In the same way $\left(\frac{m-x}{m+x}\right)^{\frac{1}{x}}=e^{-2/m}$ when $x=0$. Hence $u=e^{2/m}+e^{-2/m}$.

Also solved by *J. SCHEFFER.*

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

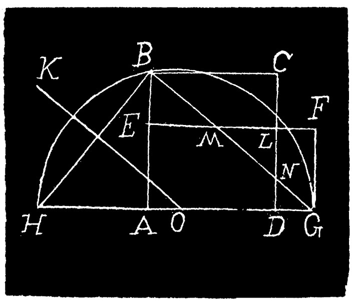
SOLUTIONS OF PROBLEMS.

69. Proposed by WILLIAM SYMMONDS, M. A., Professor of Mathematics and Astronomy in Pacific College, Santa Rosa, California; P. O., Sebastopol, California.

To divide a square card into right-lined sections in a manner, that a rectangle of a given breadth can be formed from the sections; likewise, form a square from a rectangular card.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

(1). Let $ABCD$ be the square. Produce DA to H making AH equal the given width of the rectangle, join HB, and draw KO perpendicular to HB at its mid-point, then O is the center of the circle through HB. Produce AD to meet circle at G; AG is the length of the required rectangle. Take $AE=AH$ and complete the rectangle $AEFG$.



Now the right triangle AHB=right triangle BCN=right triangle MFG.

$\therefore CN=AE$ and $DN=BE$;

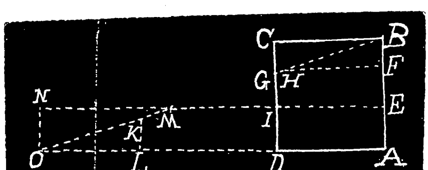
$\therefore \triangle BEM=\triangle DNG$.

$\therefore ABCD=ADNME+BCN+BEM$
$=ADNME+MFG+NDG=AEFG$.

(2). Let $AEFG$ be the given rectangle. Produce GA to H making $AH=AE$. Upon HG describe the semi-circle. Then AB is a side of the required square. Complete the square $ABCD$ and draw BG. The rest of the proof is the same as above.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

(1). Let $ABCD$ represent the square card. From A lay off on AB the the width of the rectangle successively as many times as possible, as AE, EF.



Then from the opposite corner C, lay off *one width only* of the rectangle, as CG. Now cut through on line GB. Then cut FH and EI parallel to DA.

Then will $EFGHI$ coincide with $DIMKL$, FBH with LKO, and BCG with MNO; and we have the rectangle $AENO$, of the given width AE, equivalent to the square $ABCD$.

(2). Let $AENO$ represent a rectangular card. Find the side of a square equivalent to the given rectangle. (Any geometry will show construction.)

Now from A lay off on OA the side of the square successively as many times as possible, as AD, DL. From N, lay off $NM=AD$.

Now, cut through on line OM. Then cut LK and DI perpendicular to AO.

Then will the sections of rectangle form a square as shown in first part of solution. The proof is evident.

III. Solution by C. H. WILSON, New York, New York.

Let $ABCD$ be the given square, and EF the breadth of the required rectangle. Find GH, a third proportional to EF and AB.

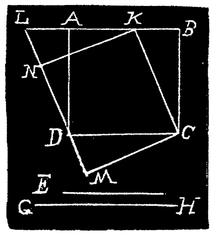


Fig. 1.

With C as a center and radius equal to GH, describe an arc cutting AB at K. (If $GH<BC$, use EF as radius). Cut off the triangle CBK and attach it in the position DAL. Draw KN perpendicular to LD. Cut of $\triangle LKN$ and attach it in the position DCM. $NKCM$ is the required rectangle, since it is equivalent to the square, and its area equals $NM\times NK=GH\times NK$. But by construction $GH\times EF=\overline{AB}^2$. $\therefore EF=NK$ (Ax).

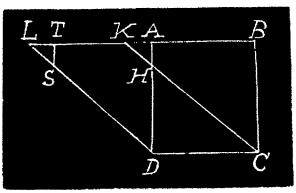


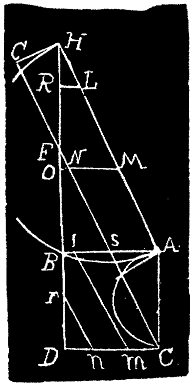
Fig. 2.

If K falls beyond A, attach $ARCB$ in position $TSDA$, cut off KAR and attach it in position LTS.

Then proceed as before.

Reverse operation (Fig. 1.) by finding mean proportional between MN and MC. With it as a radius and C as a center, describe an arc cutting MN in D. Cut off $\triangle MDC$ and attach it as NLK. Draw DA perpendicular to KL, cut off $\triangle LAD$ and attach it as KBC.

IV. Solution by the PROPOSER.



On AC, a side of the given square $ABCD$, draw a semi-circle AEC. Measure from A the chord AE equal to the given breadth of the rectangle. Produce CE, marking AB in S. On CD take CM, MN, etc., equal to AS.

Through m, n, etc., draw ml, nr, etc., parallel to CS. These will trace the lines of division required.

From the figure it is plain the rectangle $AEGH$ can be formed by joining fragments, AEC, AES, SM, m, Drn.

(2). On one of the longer sides AH of the given rectangle $AEGH$ describe the semi-circle ABH.

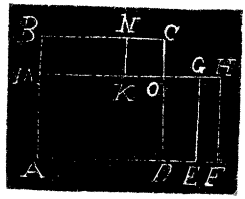
From A lay off chord AB equal in length to side of required square.

Join BH. Take BO, OR, etc., equal to AB. Through O, R, etc., draw OM, RL, etc., parallel to BA, marking the lines of division MN, LR, etc.

Hence the square $ABCD$ forms the parts of the rectangle, AES, SM, MR, RLH, HGF.

COROLLARY. When AS is greater than AB, or conversely, when AB is less than AS, the construction is quite simple.

V. Solution by J. W. SCROGGS, Principal of Rogers Academy, Rogers, Arkansas.



Let $ABCD$ be the given square, $AB=a+b$, $AM=MK=OD=GE=a$, $OK=OC=DE=b$, and $EF=b/r$.

Then area of $ADOM=a^2+ab$, and area of $BNKM=ab$.

$\therefore$ Their sum$=a^2+2ab$.

Let $b/a=r$. Then $a=br$, and area of $EFHG=b/r\times br=b^2$. $\therefore$ $AFHM=ABCD$.

CALCULUS.

Conducted by J. M, COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

49. Proposed by B. F. BURLESON, Oneida Castle, New York.

Find (1) in the leaf of the strophoid whose axis is a the axis of an inscribed leaf of the lemniscata, the node of the former coinciding with the crunode of the latter. Find (2) in a leaf of the lemniscata whose axis b the axis of a of an inscribed leaf of the strophoid, the node of the former also coinciding with the crunode of the latter.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

CASE I.

The equation of strophoid with origin at node is,

$$y^2=\frac{x(x-a)^2}{2a-x}\dots\dots\dots\dots(1).$$

The equation of lemniscate with origin at crunode is,

$$(x^2+y^2)^2=b^2(x^2-y^2)\dots\dots\dots\dots(2).$$

In order that the latter may be inscribed in the former we must have tangency. $\therefore$ x, y, and dy/dx must be equal for each curve.